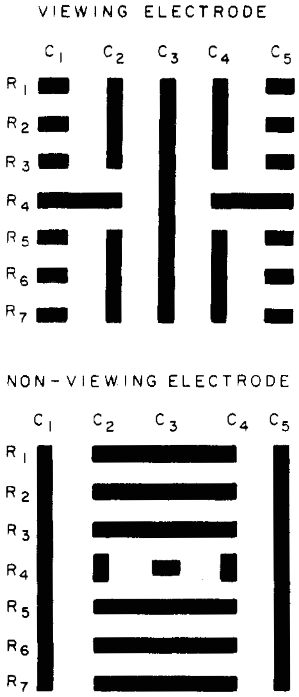

Fig. 2. Electrode pattern layout electrically equivalent to pigment island layout of Fig. 1.

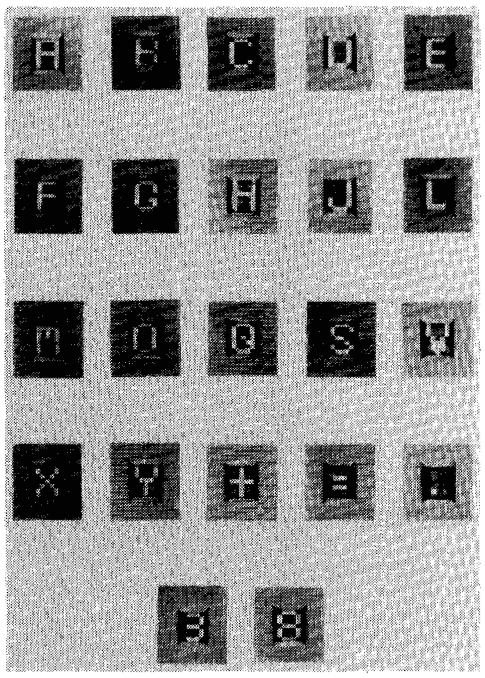

Fig. 3. Variety of characters generated by MLVS scheme.

Acknowledgment

The author would like to thank R. Liebert for the suggestion of using pattern electrodes and for carefully checking the alphanumeric voltage code as it was developed.

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Bandgap Narrowing in Silicon Bipolar Transistors

J. W. SLOTBOOM AND H. C. DE GRAAFF

Abstract—Martinelli [1] recently reported on measurements of the *I-V* characteristics of silicon bipolar transistors as a function of temperature. His conclusion was that there was no evidence of bandgap narrowing in the transistors.

Our experiments [2] on n-p-n transistors indicate that the bandgap does narrow for impurity concentrations above $N = 10^{17}$ cm^{-3}. The reason for this discrepancy follows from Martinelli's assumption that the temperature dependence of the minority carrier mobility in the p-type base is given by $T^{-2.6}$, independently of the impurity concentration, which is not justified by our measurements.

Optical absorption measurements [3], [4] have shown that the bandgap of silicon changes for high impurity concentrations. Using these measurements Kauffman and Bergh [5] and Buhanan [6] interpreted their measurements of the temperature dependence of the *I-V* characteristics and the current gain in bipolar transistors by assuming a different bandgap in the base and in the heavily doped emitter. Because of the importance of this effect for the understanding and optimal design of semiconductor devices, there has been a discussion about the presence and magnitude of this phenomenon. In a recent publication [1] dealing with the temperature dependence of the (I_B-V_{EB}) and (I_C-V_{EB}) characteristics of silicon bipolar transistors Martinelli came to the following conclusions:

1) the (I_B-V_{EB}) characteristics are nonideal and therefore cannot be used to prove the presence of bandgap narrowing.
2) the (I_C-V_{EB}) characteristics are well described by the classical model without bandgap narrowing in the base region (V_{go} = 1.20 eV).

In view of these conclusions it seems appropriate to present a short survey of arguments demonstrating why in our opinion, bandgap narrowing indeed is present. These arguments are taken from some recent publications [2] describing experiments on a number of n-p-n transistors, varying in base-doping concentration from 4×10^{15} to 2×10^{19} cm^{-3}. It appeared that bandgap narrowing (ΔV_{go}) is present for impurity concentrations above about $N = 10^{17}$ cm^{-3} and given by

$$\Delta V_{go}(N) = 9 \left\{ \ln\left(\frac{N}{10^{17}}\right) + \sqrt{\left(\ln\left(\frac{N}{10^{17}}\right)\right)^2 + 0.5} \right\} \text{ [mV]}. \tag{1}$$

Concerning the above mentioned conclusions [1], we completely agree with the first one and in fact it was for that reason that our experiments were concentrated on the (I_C-V_{EB}) instead of the nonideal (I_B-V_{EB}) characteristics. We cannot agree with

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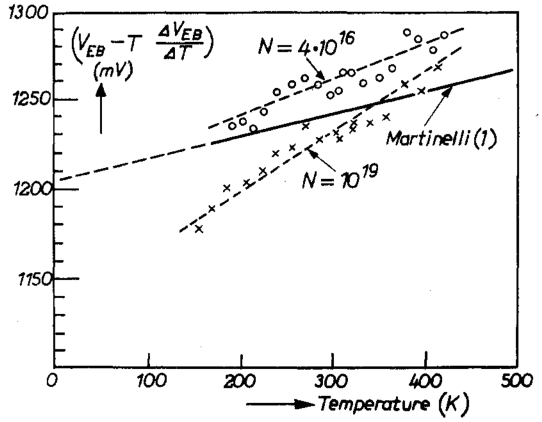

Fig. 1. Measurements of V_{EB}-$T\Delta V_{EB}/\Delta T$ as function of temperature for two transistors with different base doping concentrations [2], compared with the behavior suggested by Martinelli [1].

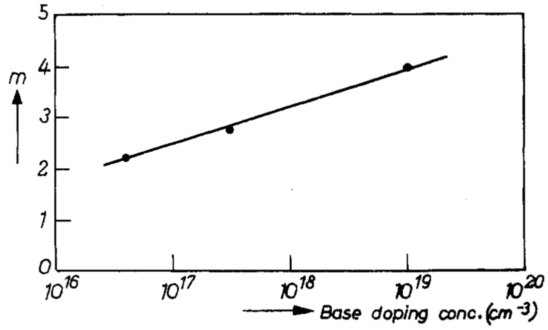

Fig. 2. The impurity concentration dependence of m (see (1)) derived from measurements [2].

his second conclusion. For transistors with base doping concentration of about $N = 10^{18}$ cm^{-3}, as were used by Martinelli, a bandgap narrowing of 42 mV would follow from (1). The reason for the disagreement lies in Martinelli's assumption that the temperature dependence of the electron mobility is given by $T^{-2.6}$, independently of the base doping concentration. This assumption means that in the expression

$$I_C = CT^m \exp\left(-q(V_{go} - V_{EB})/kT\right) \tag{2}$$

m has a constant value 1.4 for n-p-n transistors [1]. It will be shown, however, that our experiments do not agree with this assumption. Differentiation of (2) with respect to the temperature T while I_C is kept constant gives:

$$V_{EB} - T\frac{dV_{EB}}{dT} = V_{go} + m\frac{kT}{q}. \tag{3}$$

We measured V_{EB} as a function of temperature for a number of n-p-n transistors with different impurity concentrations in the base while I_C was constant. In Fig. 1 the term $(V_{EB} - T\Delta V_{EB}/\Delta T)$, which was directly derived from these measurements is shown as a function of temperature for two transistors with different base doping concentrations. For comparison a line with $V_{go} = 1.205$ eV and $m = 1.4$ is included. It is clear that this line does not fit our experiments very well, these experiments indicating that m should be a function of the impurity concentration rather than being a constant. These transistor temperature measurements are not enough to derive the value of m and the bandgap narrowing accurately and we combined them with similar temperature measurements of the base sheet resistance ($R_{\square \text{Base}}$) underneath the emitter, which was taken from the same slice as the transistor [2]. The m-values derived in this way are shown in Fig. 2 (the β-values from [2c] table 2 are the same as the

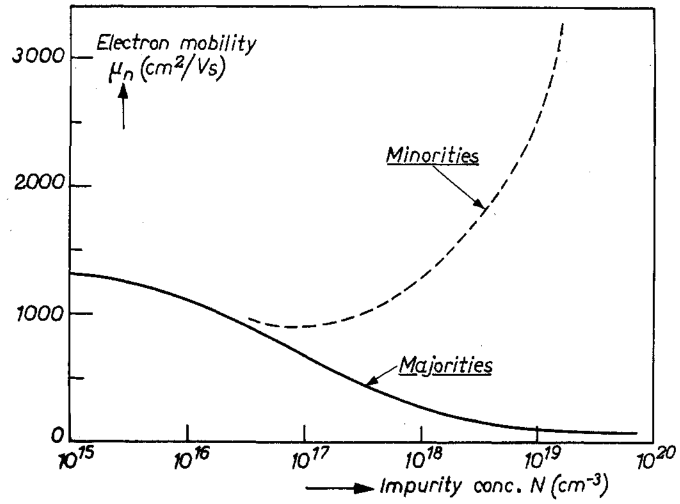

Fig. 3. Comparison of the majority carrier mobility with the minority carrier mobility as a function of impurity concentration, assuming that there is no bandgap narrowing at all (see (4)).

m-values). We see that for $N = 10^{18}$ cm^{-3} m is about 3. When this value for m is applied in Martinelli's [1, fig. 6], which gives a relationship between m and the bandgap for his transistors, a narrowing of about 45 mV appears. This accords well with the bandgap narrowing of 42 mV which we predict according to (1).

It is important to notice that the bandgap narrowing was derived not only from the temperature dependence as described above, but also from the magnitude of the I_0 and $R_{\square \text{Base}}$ measured at room temperature [2]. Bandgap narrowing values obtained by both methods agree and are fitted by (1). It was pointed out in [2c] that in all these transistor measurements it is in fact the product $(\mu_n n_{ie}{}^2)$ which is measured and that the behavior of the pn-product, $n_{ie}{}^2$, can only be derived from these measurements if the minority carrier mobility is known. Unfortunately, there are no experimental data on minority carrier mobility for high impurity concentrations. For concentrations below 10^{17} cm^{-3} there is no difference between drift mobility and conductivity mobility [7], and although there is no experimental evidence for high impurity concentrations we assumed that the electron mobilities in n- and p-type silicon are similar as a function of impurity concentration and temperature. A number of arguments support this assumption:

1) Theoretical calculations [8] of the pn-product as a function of temperature and impurity concentration.
2) If the measurements were interpreted in terms of minority carrier mobility and it was assumed that no bandgap narrowing at all occurred, the resultant mobility behavior would seem highly improbable, being given by

$$\mu_{n,\text{min.}}(N,T) = \mu_{n,\text{maj.}}(N,T) \exp\left(q\Delta V_{go}(N)/kT\right) \tag{4}$$

as shown in Fig. 3.
3) Using bandgap narrowing values according to (1) in calculations for the magnitude and temperature dependence of injection of minority carriers into heavily doped regions, such as n^+ or p^+ emitters, buried layers, isolation regions etc. agrees quantitatively well with measurements [9], [10].

In conclusion it can be said that Martinelli's assumption that the parameter m in (2) should have a constant value of 1.4 (for bipolar n-p-n transistors) is not in agreement with our experiments, which show that m has a much higher value and varies with the impurity concentration in the base region.

It has been shown that our interpretation in terms of bandgap narrowing, which explains our own measurements, is equally applicable to Martinelli's measurements and indicates that for base doping concentrations of about $N = 10^{18}$ cm^{-3} a bandgap narrowing of about 42 mV occurs. Several arguments supporting our assumption concerning the minority carrier mobility have been discussed.

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Precise State Control of AC Gas-Discharge Displays

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***Abstract*—Erasing and writing techniques which change the state of cells very precisely with wide cell and waveform tolerances are presented. As a consequence of this precision, a display may be designed with wider cell and pulse tolerances and yet require no recovery time between state changes.**

Techniques are presented for erasing and writing cells of ac gas-discharge display–storage panels which change the state of the cells with very high precision over a large range of cell characteristics and pulse amplitudes and widths. Consequently, little or no recovery time is required between state changes in a particular cell, permitting fast updating of displayed information without individual adjustment of the erase and write amplitudes of each display. These techniques use pulse amplitudes which are identical to those required by recently described nondestructive cursor and fast light-pen tracking techniques [1]. Since relatively high peak voltages would be required for the cursor and light-pen functions, the use of the same voltages to perform erasing and writing would reduce the cost of the display system and provide faster updating as well.

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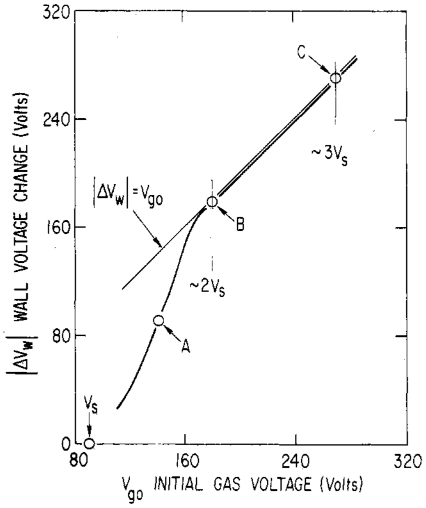

Fig. 1. Typical measured voltage transfer curve of an ac gas display cell. Point "A" is the locus of "perfect" conventional write and erase discharges. Points "B" and "C" are the loci of nominal linear write and erase discharges. Point "B" is also the locus of nominal "on" state sustaining discharges.

The object of the erase operation is to form discharges in the selected cells which transfer exactly the wall voltage stored by the last sustaining discharge, thus reducing the wall voltage to zero and extinguishing the glow in one pulse period. One conventional type of erase pulse has a smaller amplitude than the sustain pulses and is wide enough to allow the voltage transfer to go to completion. The ideal erase discharge of this type is represented on the measured voltage transfer curve [2], [3] of Fig. 1 as point "A" on the steepest region where the slope is 3 to 4, so the accuracy of the erase operation is highly sensitive to variation of pulse amplitude and cell characteristics. Other conventional erase techniques also have critical parameters, such as pulse width.

After imperfect erase pulses, many sustain cycles may be required to reduce the residual wall voltage to zero. Even after some 100 sustain cycles following an erase operation, the authors have observed a dependence of the ideal write amplitude upon the preceding erase amplitude. The time required to perform this "clean-up" operation can slow down the panel update time. This can be a significant limitation in large display panels used for graphics. One way the designer can minimize or avoid this clean-up time is by tightening the margins on the pulse amplitudes and panel cell characteristics and individually adjusting the pulse amplitudes of each display for proper operation. Such adjustments may be required repeatedly. These expedients add materially to the product cost. We present below a new way of erasing and writing which avoids such expedients. The same ideas are embodied in a patent by Petty and Liddle [4], but appear not to have been discussed in the literature.

The new erase technique uses pulses which are wide enough for complete voltage transfer and large enough in amplitude so that the discharges occur in the region of the voltage transfer curve at and above the normal sustain point. The new erase pulse is shown in Fig. 2(a) as it appears across the electrodes of a selected cell, imbedded in a rectangular sustain-pulse series of amplitude V_s. Fig. 2(b) and (c) show wall voltages for cells which are initially "on" and initially "off," respectively. In the case of a cell initially "on," the initial wall voltage is nearly V_s at the time the erase pulse rises to $2V_s$. The inital gas voltage of the erase discharge is the sum of the wall and applied voltages, nearly $3V_s$. The voltage transfer for discharges at and above the sustain point is virtually equal to the peak gas voltage, so the wall voltage be-